

In SI units

$$c = 2.99792458 \times 10^8 \text{ m/s} \quad (\text{exact})$$

$$\hbar = 1.0545717 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$\epsilon_0 = 8.854187817 \times 10^{-12} \text{ F}\cdot\text{m}^{-1} = \frac{1}{\mu_0 c^2} \text{ with } \mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$$

$$1 \text{ GeV} = 1.6021765 \times 10^{-10} \text{ J}$$

a) Dimensionally $L = \frac{L}{T} \times \frac{T \cdot E}{E} = \frac{\hbar c}{E}$

Natural unit of length is GeV^{-1}

$$1 \hbar c / \text{GeV} = \frac{1.05457 \times 10^{-34} \text{ J}\cdot\text{s} \times 2.99792 \times 10^8 \text{ m/s}}{1.60217 \times 10^{-10} \text{ J}} = 1.97327 \times 10^{-16} \text{ m}$$

b) Area is length squared, unit is GeV^{-2}

$$1 (\hbar c / \text{GeV})^2 = 3.89379 \times 10^{-32} \text{ m}^2$$

c) $T = \frac{T \cdot E}{E} = \frac{\hbar}{E}$

time is in GeV^{-1}

$$1 \hbar / \text{GeV} = 6.58212 \times 10^{-25} \text{ sec}$$

d) Energy $[E] = \left[\frac{Q^2}{\epsilon_0 L} \right]$ so $[E_0] = \left[\frac{Q^2}{\epsilon_0 L} \right]$ & $Q = \sqrt{\epsilon_0 E L}$

Potential $[V] = \frac{E}{Q} = \sqrt{\frac{E}{\epsilon_0 L}} = \sqrt{\frac{E L}{\epsilon_0 \hbar c}} = \left[\frac{E}{\sqrt{\epsilon_0 \hbar c}} \right]$

so V is in GeV .

$$1 \text{ GeV} / \sqrt{E_0 \hbar c} = 3.02822 \times 10^8 \text{ V}$$

e) Charge $[Q] = \sqrt{E_0 \hbar c} = \sqrt{E_0 \hbar c}$

charge is dimensionless in natural units

$$1 \sqrt{E_0 \hbar c} = 5.29082 \times 10^{-19} \text{ C}$$

f) $1 \text{ m}^2 = 10^4 \text{ cm}^2 = 10^{28} \text{ b} = 10^{31} \text{ mb}$

$$\begin{aligned} 1 (\hbar c)^2 \text{ GeV}^{-2} &= 3.89379 \times 10^{-32} \times 10^{31} \text{ mb} \\ &= 0.389379 \text{ mb} \end{aligned}$$

g) $e = 1.602177 \times 10^{-19} \text{ C}$

$$= \frac{1.602177 \times 10^{-19}}{5.29082 \times 10^{-19}} \text{ natural units}$$

$$= 0.302822$$

Inverse conversion factors: $1 \text{ m} = \frac{1}{1.97327 \times 10^{-16}} \frac{\hbar c}{\text{GeV}} = 5.0677 \times 10^{15} \frac{\hbar c}{\text{GeV}}$

$$1 \text{ m}^2 = \frac{1}{3.89379 \times 10^{-32}} = 2.5682 \times 10^{31} \frac{\hbar c^2}{\text{GeV}^2}$$

$$1 \text{ s} = \frac{1}{6.58212 \times 10^{-25}} = 1.51927 \times 10^{24} \frac{\hbar}{\text{GeV}}$$

$$1 \text{ volt} = 3.30227 \times 10^{-9} \text{ GeV} / \sqrt{E_0 \hbar c}$$

$$1 \text{ C} = 1.890066 \times 10^{18} \sqrt{E_0 \hbar c}$$

$$1 \text{ mb} = 2.5682 \frac{\hbar c^2}{\text{GeV}^2}$$