(a) The transformed fields are

$$\begin{pmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \end{pmatrix} = \left(e^{-i\sum_a \omega_a \sigma_a/2}\right) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{i}{2}\sum_a \omega_a \sigma_a\right] \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} + O(\omega^2).$$

$$\therefore \sum_a \omega_a D_a \phi_j = -\frac{i}{2}\sum_{a,j'} \omega_a (\sigma_a)_{jj'} \phi_{j'} .$$

$$\therefore D_a \phi_j = -\frac{i}{2}\sum_{j'} (\sigma_a)_{jj'} \phi_{j'} .$$

$$\therefore D_a \phi_j^+ = \frac{i}{2}\sum_{j'} (\sigma_a)_{jj'}^* \phi_{j'}^+$$

As a matrix $\sigma_a = \sigma_a^+$, so $(\sigma_a)_{jj'} = (\sigma_a)_{j'j}^*$

$$\therefore D_a \phi_j^+ = \frac{i}{2}\sum_{j'} \phi_{j'}^+ (\sigma_a)_{j'j} .$$

It's a good idea to verify $D_a \mathcal{L} = 0$, from

$$\begin{aligned} D_a \phi^+\phi &= \sum_j D_a (\phi_j^+ \phi_j) \\ &= \sum_j \left[(D_a \phi_j^+) \phi_j + \phi_j^+ D_a \phi_j \right] \\ &= \frac{i}{2}\sum_{jj'} \left[\phi_{j'}^+ (\sigma_a)_{j'j} \phi_j + \phi_j^+ (\sigma_a)_{jj'} \phi_{j'} \right] \\ &= 0. \end{aligned}$$

(b) $$J_a^\mu = \sum_j \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_j} D_a \phi_j + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_j^+} D_a \phi_j^+ \right]$$

$$= -\frac{i}{2} \sum_{jj'} \left[-\partial^\mu \phi_j^+ (\sigma_a)_{jj'} \phi_{j'} + \phi_{j'}^+ (\sigma_a)_{j'j} \partial^\mu \phi_j \right]$$

$$= -\frac{i}{2} \left(-\partial^\mu \phi^+ \sigma_a \phi + \phi^+ \sigma_a \partial^\mu \phi \right)$$

$$Q_a = \int d^3\vec{x} \; J_a^0$$

$$= \frac{i}{2} \sum_{jj'} (\sigma_a)_{jj'} \int d^3\vec{x} \left(\phi_j^+ \dot{\phi}_{j'} - \dot{\phi}_j^+ \phi_{j'} \right)$$

$$= \frac{i}{2} \sum_{jj'} (\sigma_a)_{jj'} \int d^3\vec{x} \left(\phi_j^+ \pi_{j'}^+ - \pi_j \phi_{j'} \right),$$

where π_j & π_j^+ are the canonical momentum fields for ϕ_j & ϕ_j^+, e.g.

$$\pi_j = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_j} = \frac{\partial}{\partial \dot{\phi}_j} \left(-\partial \phi^+ \cdot \partial \phi - m^2 \phi^+ \phi - \lambda (\phi^+ \phi)^2 \right)$$

$$= \dot{\phi}_j^+$$

$$\pi_j^+ = \dot{\phi}_j$$