$$\psi = \int \frac{d^3\vec{p}}{(2\pi)^3} \, a_p \, e^{-iE_p t + i\vec{p}\cdot\vec{x}}$$

with $E_p = p^2/2m$.

$$\tilde{G}_2(p_1, p_2) = \int d^4x \, d^4y \, e^{ip_1\cdot x + ip_2\cdot y} \times$$

$$\times \Big[\theta(x^0 \geq y^0) \langle 0 | \psi(x) \psi^\dagger(y) | 0 \rangle$$

$$+ \theta(y^0 > x^0) \langle 0 | \psi^\dagger(y) \psi(x) | 0 \rangle \Big]$$

$$= \int d^4x \, d^4y \int \frac{d^3\vec{p} \, d^3\vec{q}}{(2\pi)^6} \, e^{ip_1\cdot x + ip_2\cdot y - ip\cdot x + iq\cdot y} \, \theta(x^0 \geq y^0)$$

$$\times \underbrace{\langle 0 | a_p \, a_q^\dagger | 0 \rangle}_{=(2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q})}$$

$$= \int d^4x \, d^4y \int \frac{d^3\vec{p}}{(2\pi)^3} \, e^{ip_1\cdot x + ip_2\cdot y - ip\cdot(x-y)} \, \theta(x^0 \geq y^0)$$

$$= \qquad '' \qquad e^{i(p_1 - p)\cdot(x-y) + i(p_1 + p_2)\cdot y} \, \theta(x^0 \geq y^0)$$

$$= \int d^4y \, e^{i(p_1+p_2)\cdot y} \int_0^\infty dz^0 \int d^3\vec{z} \int \frac{d^3\vec{p}}{(2\pi)^3} \, e^{i(p_1^0 - E_p) z^0} \, e^{-i(\vec{p}_1 - \vec{p})\cdot\vec{z}}$$

where $z = x - y$

$$= (2\pi)^4 \delta^{(4)}(p_1 + p_2) \int \frac{d^3\vec{p}}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{p}_1 - \vec{p}) \int_0^\infty dz^0 \, e^{i(p_1^0 - E_p) z^0}$$

$$= (2\pi)^4 \delta^{(4)}(p_1 + p_2) \, \frac{i}{p_1^0 - \vec{p}_1^{\,2}/2m + i\epsilon} \, .$$