Homework 1.2 Physics 419

Due: Sept. 10

1. (a) A rock of mass $m$ is whirling around on the end of a string, initially in a circle of radius $r_0$ at angular velocity $\omega_0$. The string passes down a hole drilled through the center of a stationary cylinder, and you are holding onto the other end. At time zero you begin to pull the string down at a steady rate $u$. What is the angular velocity of the rock as a function of time? First step: identify what is (and perhaps what is not) conserved in this situation.

   (b) Suppose instead that the string is fixed to the outside of the cylinder and it becomes wound on it like thread on a spool as the rock goes around and around. What is conserved in this case? Explain carefully.

   (c) (optional, but give it a try at least) In this second case, find the motion as a function of time. A convenient form is the angular position $\theta(t)$ of the point at which the string comes out of contact with the spool. (suggested notations and conventions: cylinder radius $R$, initial length of free string $r(0)$, $\theta(0) = 0$ and speed of rock $v$)

2. The center of mass of a system may be computed in stages. That is, split the system $S$ two parts $S_1$ and $S_2$ of total mass $M_1$ and $M_2$ respectively, replace each subsystem by a particle carrying the total mass of the subsystem placed at its center of mass, and finally compute the center of mass of this artificial two particle system. Demonstrate from the definitions that this procedure is valid.

3. This problem is essentially the same as MT 2-56, except that I changed the numbers to be mean. A lunar lander needs to hover above the surface of the moon to effect a rescue. The gravitational acceleration at the moon's surface is about $g/6$. If the exhaust velocity is 2500 m/s and the amount of fuel which can be expended before the effort must be called off is 15% of the total mass of the lander, how long can it hover?

4. Undoubtedly you have already done the exercise in section 1.4.3 of the notes. Copy it over neatly, including the steps which are already there, and hand it in. More explicitly, using the strong form of Newton's third Law, show that $d\mathbf{L}/dt = \mathbf{N}_{\text{ext}}$.

5. In class, we examined the stability of the Ringworld to motion in its plane. Determine whether it is stable to a slight displacement *perpendicular* to the plane in which it lies.

6. (a) Using the flux theorem, compute the gravitational field due to an infinite cylinder of radius $R$ and mass density $\rho$. (both inside and outside the cylinder).

   (b) Attempt to compute the gravitational potential field instead. Do you encounter a problem? What do you make of that?

7. Determine which of these planar force fields is conservative (a picture is a good idea):

   (a) $\mathbf{F} = (y\hat{\mathbf{x}} - x\hat{\mathbf{y}})/r^2$

   (b) $\mathbf{F} = \mathbf{a}h(\mathbf{a} \cdot \mathbf{r})$ ($\mathbf{a}$ is a fixed vector and $h$ is a differentiable function)

8. A projectile (mass $m$) is fired at 45° above the horizontal with an initial kinetic energy $E_0$. At the top of its trajectory, it explodes into two fragments of masses $m_1$ and $m_2$. The explosion releases an additional energy $E_0$ and the first fragment travels straight downward. Find the velocity and initial direction of motion of the second fragment as a function of the masses and $E_0$. There is a maximum value of $m_1/m$ such that the described situation is possible. What is that ratio? (ignore air resistance for this problem of course)