

Due: Oct. 25

**1. matrix manipulations.** Find the eigenvalues and eigenvectors of the following matrices.

$$(a) : \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (b) : \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

**2. strongly coupled oscillators.**

Discuss the limit  $\kappa_3 \gg \kappa_1 \gg \kappa_2$  for the pair of coupled harmonic oscillators we discussed in class. Expand the square root in equation (6.29) in the notes to lowest order in  $(\Delta\kappa/\kappa_3)^2$  to get the two normal mode frequencies and describe roughly what these modes look like. Explain as you do why one frequency is very large and the other not.

**3. double pendulum.**

This problem is concerned with the double pendulum discussed in section 2.4 of the Notes (p. 32). Take  $m_1 = m_2 = m$  and  $\ell_1 = \ell_2 = \ell$  for simplicity, and assume that everything is free to rotate completely around without getting tangled up.

(a) Identify all the equilibria for the potential of this system (there are four), and determine their type in terms of the number of stable and unstable directions at each.

(b) Linearize the equations of motion about the stable equilibrium and compute the normal modes and their frequencies.

**4. determinant is volume (not to be handed in).** If we consider a matrix as defining a linear transformation on  $n$ -dimensional Euclidean space with the usual volume element, and with an orthogonal basis, then the determinant is the volume spanned by the columns of the matrix, viewed as vectors themselves. You can easily establish that for the two- and three-dimensional cases.

(a) Show explicitly By explicitly computing the determinant of a general  $2 \times 2$  matrix  $A$ , show that it is the area of the box spanned by its columns. Do the same for a  $3 \times 3$  matrix (so now volume for area).

(b) Realize that the columns of the matrix of  $\mathbf{A}$  are the images  $\mathbf{A}\hat{\mathbf{e}}_a$  of the basis vectors. Using that, convince yourself that the determinant is the factor by which  $\mathbf{A}$  stretches volumes.